JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 5, September–October 1995

# Precision Position Control of Piezoelectric Actuators Using Charge Feedback

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**This paper examines the current state of the art in position control of actuators constructed from piezoelectric materials. Piezoelectric actuators are typically activated by an applied voltage, but there exists significant hysteresis between the voltage applied to the actuator and the actuator output. An explanation for this hysteresis is presented using as a basis the laws governing the behavior of dielectric materials. This hysteresis explanation indicates that it is impossible to derive a linear relationship between an input voltage and actuator output. In light of this, a new control relationship is developed based on the piezoelectric material constitutive relationships that relates actuator displacement to an amount of applied charge. Experimental data from tests of both voltage control and charge control show that charge control is significantly more linear and less hysteretic than voltage control over the same actuator displacement range.**

## Nomenclature

$A$ = electrode area, m$^2$
$C$ = series capacitance, F
$D$ = electric displacement, C/m$^2$
$d, g$ = piezoelectric constants
$E$ = total electric field, V/m
$n$ = number of actuator layers
$P$ = polarization, C/m$^2$
$Q_f$ = free charge, C
$Q_t$ = total free charge, C
$S$ = material strain
$s^D$ = elastic compliance at constant electric displacement, m$^2$/N
$s^E$ = elastic compliance at constant total field, m$^2$/N
$T$ = stress, Pa
$t$ = layer thickness, m
$V$ = applied voltage, V
$V_{\text{in}}$ = amplifier input voltage, V
$Y$ = Young's Modulus, Pa
$\Delta l$ = actuator deflection, m
$\varepsilon^T$ = permittivity at constant stress, C$^2$/N-m$^2$
$\rho_f$ = free charge density, C/m$^2$

## Introduction

THE extensive studies carried out on piezoelectrics in the aftermath of World War II for sonar, acoustic, and accelerometer applications have provided a technological base for the recent interest in the use of piezoelectric materials as actuators and motors. Much of the technology developed in these early studies is directly applicable to the use of piezoactuators in acceleration and velocity control, but transferring this technology and applying it to position control should be examined with a more critical eye. By its very nature, the study of vibroacoustics is more concerned with velocities and accelerations than absolute positioning, and so methods developed for motion and force control may not be appropriate for precision position control.

The issue of precision position control of piezoelectric actuators is critical if this technology is to be applied in increasingly demanding applications. In one particular application, the NASA National Advanced Optics Mission Initiative project (NAOMI) [formerly Space Laser Energy (SELENE)[1]], piezoelectric actuators have been proposed as the pointing and focusing elements for thousands of small mirror-lenslets because of their potential for miniaturization and the absence of moving parts. The positions of these actuators must be precisely controlled both statically and dynamically to the submicron level and across wide frequency ranges. This requirement necessitates a careful study of the concept and design of the driving electronics of the system. This paper is focused on finding an appropriate method for driving piezoelectric actuators for ultraprecision position and motion control.

This paper begins with a derivation of a control law that is typical for piezoelectric material-based actuators. This derivation will then be critically examined and some assumptions that compromise the accuracy of the model in terms of applicability to position control identified. A new model for precise position control of piezoelectric actuators will then be derived. The accuracy of this model will then be demonstrated experimentally.

## Model Development

The general form of the piezoelectric equations can be written in compressed tensor form as[2]

$$S_p = s_{pq}^E T_q + d_{kp} E_k \tag{1}$$

$$D_i = d_{iq} T_q + \varepsilon_{ik}^T E_k \tag{2}$$

The typical fundamental element of a piezoelectric material-based actuator is a flat wafer of material with electrodes distributed over the top and bottom of the wafer. Single elements of this type have been used to apply bending moments to beam elements to perform shape control and vibration suppression, and linear actuators are constructed by stacking a number of these elements together (see Fig. 1). It is typical in these wafer piezoceramic elements to have the three-direction of the piezoceramic (the material

Received Oct. 13, 1994; revision received May 1, 1995; accepted for publication May 15, 1995. 

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polarization direction) correspond with the thickness direction of the wafer element.

The goal of this investigation is to develop a control law for delivery of a very precise displacement or force with a piezoelectric material-based actuator. Therefore, the need is for some sort of way to command this force or displacement. The logical place to start this investigation is with the current state of the art in control of piezoelectric actuators. As an example, the derivation of control equations for a piezomaterial-based actuator will be performed in a fashion that is currently typical.

An excellent example of a piezoelectric material-based actuator is a stack actuator, which consists of a number of wafer elements bonded together mechanically in series and electrically in parallel. Forces and displacements are applied in the wafer three-direction (the wafer thickness direction) in a stack actuator. To develop control equations in this case, the piezoelectric constitutive relationships are generally simplified through the use of tensor compression techniques and the assumption that all loads and fields are significant only in the three-direction. These steps result in an expression that is an approximation for Eq. (1) (see Ref. 3, for example)

$$S_3 = (1/Y_{33})T_3 + d_{33}E_3 \tag{3}$$

where $Y_{33}$ is the modulus of the piezoelectric material in the three-direction.

The wafer-with-distributed-electrodes geometry of piezoelectric actuators is very reminiscent of a two-plate capacitor, and so the temptation is to make the following argument: if a voltage is applied to the electrodes, an electric field will be created in the three-direction, and this field will have a magnitude of $E_3 = V/t$. Making this substitution yields the expression

$$S_3 = (1/Y_3)T_3 + d_{33}(V/t) \tag{4}$$

This is a very convenient equation because it gives the output strain $S_3$ in terms of the stress applied to the piezomaterial $T_3$ and the voltage applied to the electrodes $V$. A convenient expression, yes, and also one that is the basis of much of the current work in piezoelectric actuators. It should be pointed out, however, that since piezoelectric materials are dielectrics, equating $V/t$ to $E_3$ is really only an approximation. The following section discusses why this approximation is not adequate for high-accuracy control applications and results in nonlinear, hysteretic stack behavior that can limit controller gains and lead to undesirable system behaviors such as limit cycles.

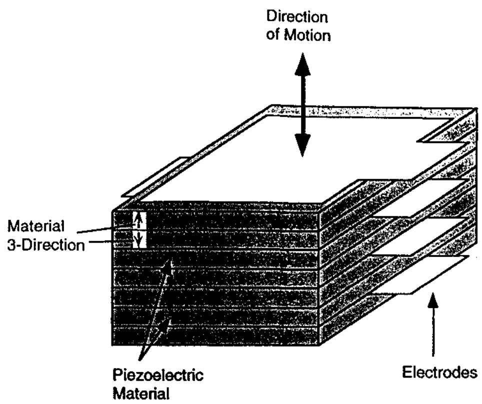


**Fig. 1 Sketch of a piezoelectric stack actuator.**

## Piezoceramics as Dielectrics

The fact is that since piezoelectrics are dielectric materials, the field applied to a dielectric, in this case $V/t$, is not the total field that is present in the material $E_3$ or, more generally, $E_k$. To better illustrate this point it is useful to digress for a moment and discuss the electromagnetic theory of dielectrics.[4]

It is helpful to first discuss the root causes of electric fields in dielectrics. They exist because of the presence of charge, and charge in dielectrics comes in two types, generally referred to as "free charge" and "bound charge." The first type, free charge, may be electrons on an electrode or ions embedded in the piezomaterial; in any case it is what is commonly thought of as electric charge. The second type, bound charge, is the result of material polarization or how much the electric dipoles in a material are aligned. Bound charge is not charge in the electrons-on-a-conductor sense. Polarization in a dielectric causes an electric field, and the bound charge is just the amount of charge that would cause an equivalent field. Both free charge and bound charge contribute to the total field in a dielectric.

Returning to the piezoelectric constitutive equations, it is noted that implicit in this model is the assumption that piezomaterials behave as linear dielectrics. A linear dielectric is a material whose polarization ($P$) is a linear function of the electric field present in the material ($E$). The relationship between these two quantities is

$$P_k = \varepsilon_0 \chi_e E_k \tag{5}$$

where $\varepsilon_0$ is the permittivity of free space, and $\chi_e$ is the susceptibility of the material. The polarization represents the bound charge contribution to the total field.

It is important to note that the electric field $E$ in Eq. (5) is the total field in the dielectric material. It may be due to polarization (a change of which causes material strain in piezoceramics), or to free charges, or a combination of the two. This relationship and the following discussion explains why there is hysteresis in voltage control of piezoelectric actuators.

Figure 2 illustrates what happens when a piece of piezomaterial is placed in an external field equal to $V/t$. Before the material is placed in the external field (Fig. 2a), there is a field present in the material ($E_0$) due to polarization. Placing the material in the field changes the total electric field (Fig. 2b). Since $E$ is coupled to $P$ through Eq. (5), this is followed by a change in the material polarization, and thus the material experiences a strain, represented in Fig. 2c as a distortion of the original circular shape into an ellipse. This change in polarization causes an additional change in the electric field ($d$), which is again followed by a change in polarization, and on and on and on. The field and polarization will eventually reach

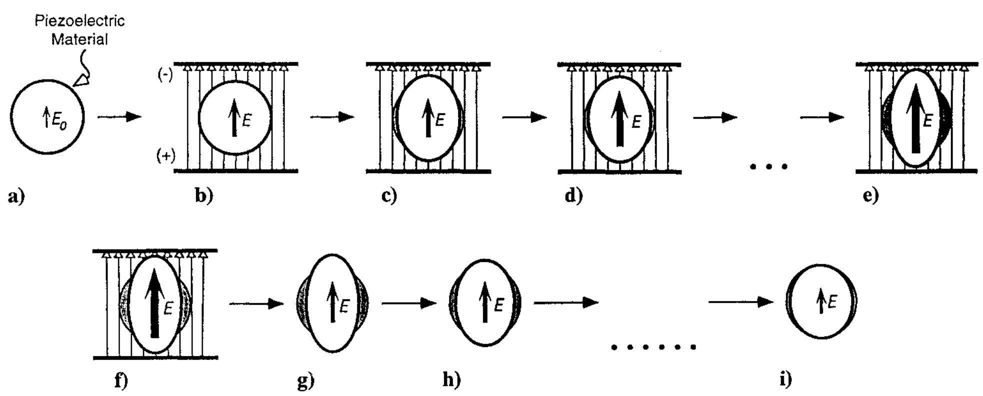


**Fig. 2 Sketch illustrating the interdependence of piezoelectric material strain, the applied electric field, and the total electric field.**

an equilibrium point (Fig. 2e) where Eq. (5) is again satisfied, but under the influence of the external field. The point is that the applied field does not simply sum with the field in the material: it triggers a complex feedback phenomenon between $P$ and $E$.

The inverse process occurs when the piece of material is removed from the field. The material starts out at equilibrium under the influence of the external field (Fig. 2f). When the material is removed from the field, there is a change in the total field (Fig. 2g), followed by a change in the polarization (Fig. 2h), and on and on until $P$ and $E$ again reach an equilibrium point (Fig. 2e). Notice that the piece of piezoelectric material in the sketch is left with some residual strain after the removal of the applied field. This is because there is no requirement, and therefore no reason to expect, that the material will return to its original state upon the removal of the applied field. The only requirement is that it returns to an equilibrium state, as expressed by Eq. (5). Since the material polarization both contributes to and is a function of the total field in the material, there exist many possible equilibrium states for a given applied field intensity. Since the strain in a piezoelectric material results directly from changes in the material polarization, the result is that there exists a range of strain values possible for every applied field value, resulting in actuator hysteresis.

An alternate control strategy is needed for wafer-based piezoactuators since there is no simple relationship between the field applied to a wafer of piezomaterial and the field actually in the material, as evidenced by the hysteresis inherent in voltage control of piezoactuators. A way around this is to eliminate the pesky $E$ term from the constitutive equations altogether and develop control equations in terms of some other controllable quantity. If Eqs. (1) and (2) are combined and $E_k$ eliminated, the expression

$$S_p = s_{pq}^D T_q + g_{kp} D_k \tag{6}$$

is found, with the new constants expressed in terms of the original ones as

$$s_{pq}^D = s_{pq}^E - d_{pk} g_{kq} \tag{7}$$

$$g_{ip} = \beta_{ik}^T d_{kp} \tag{8}$$

$$\beta_{ik}^T \varepsilon_{jk}^T = \delta_{ij} \tag{9}$$

where $\delta_{ij}$ is the Kronecker delta.

This new relationship [Eq. (6)] describes the piezomaterial strain $S_p$ in terms of the applied stress $T_q$ and the electric displacement $D_k$. The question is, however, is this new quantity $D_k$ controllable? Or, to put it another way, are we any better off than we were before?

The use of the electric displacement quantity is often avoided, likely because the concept of an electric "displacement" is not particularly intuitive. One simple way to visualize $D$ is to imagine two uniform circular smears of charge, one positive and one negative (see Fig. 3a). If these two smears of charge are placed directly on top of one another, a region of zero net charge is created (Fig. 3b). However, if the centers of the two smears are displaced, say by a distance $D$, two caps of opposite charge are formed surrounding a region of zero net charge (Fig. 3c). The field between the two caps of charge is conceptualized as a displacement $D$.

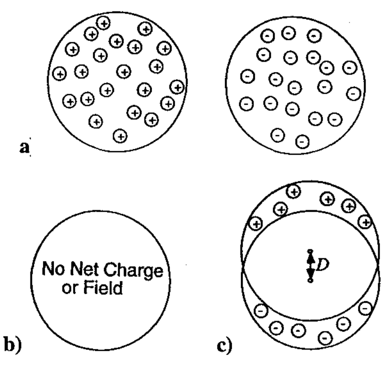


**Fig. 3 Example of electric displacement $D$.**

It appears from the sketch that the electric displacement causes a polarization in the material. It is more correct, however, to say that the material polarization $P$, the applied electric field, and the stress $T$ combine to cause the electric displacement $D$, which is really nothing more than a conceptual device to tie everything into one charge-based model.

The beauty of using the electric displacement is that it allows application of Gauss' law for dielectrics to the problem, the differential form of which can be written

$$\nabla \bullet D = \rho_f \tag{10}$$

The integral form of Gauss' law is more useful in this case and is written

$$\int_{\text{surface}} D \bullet \mathrm{d}A = Q_f \tag{11}$$

where $Q_f$ is the total free charge enclosed in the specified Gaussian volume. Note that both Eqs. (10) and (11) rely on knowledge of the free charge in the system. This is an important result because free charge in a system can be directly controlled with amplifiers and electronics, unlike the total electric field that also depends on bound charge within the dielectric. In the following section, Gauss' law is applied to determine the electric displacement and therefore find mechanical displacement control relationships for a piezoelectric material-based actuator.

## Example: Stack Actuator Control Relationship

As an example, Gauss' law will be applied to calculate the electric displacement, and therefore a control relationship, for a piezoelectric stack actuator. Typically these actuators have the general appearance shown in Fig. 4a, where the piezoelectric material is in thin wafers with electrodes bonded on the top and bottom of each wafer.

Gauss' law can be applied by looking at a single wafer with its positive and negative electrodes (see Fig. 4b). Assuming that the wafers are thin enough that field fringing at the electrode edges will not have a significant effect, a Gaussian pillbox can be drawn (Fig. 5) that extends slightly below and above the positive electrode into the piezoelectric material. Since $D_3$ is perpendicular to the lid and bottom of the pillbox, only these surfaces contribute to the calculation, and since there is no material below the positive electrode, only the

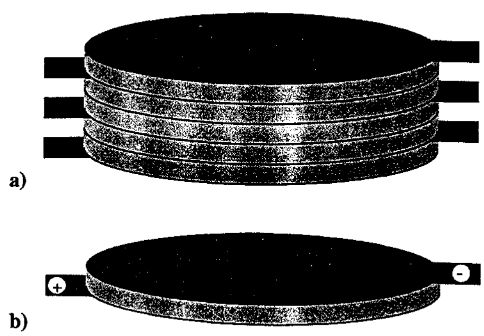


**Fig. 4 a) Sketch of a piezoelectric stack actuator and b) a single element from the stack with its positive and negative electrodes.**

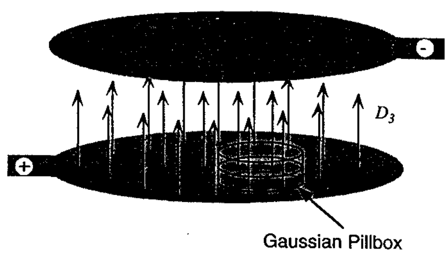


**Fig. 5 Gaussian volume used in the electric displacement calculation.**

lid contributes. Applying Eq. (11) to this geometry and integrating over the area of the positive electrode yields

$$D_3(A) = Q_f \tag{12}$$

In the case of the stack actuator, it is convenient and reasonable to assume that all of the strains, stresses, and electric displacements will be much greater in the three-direction than any other. Under these assumptions Eq. (6) can be simplified to

$$S_3 = \left(\frac{1}{Y_{33}} - \frac{d_{33}^2}{\varepsilon_{33}^T}\right)T_3 + \frac{d_{33}}{\varepsilon_{33}^T}D_3 \tag{13}$$

where the substitutions in Eqs. (7–9) have been made to return to the original choice of constants. Combining Eqs. (12) and (13) yields a control expression for a piezoelectric actuator that is based on charge, not voltage,

$$S_3 = \left(\frac{1}{Y_{33}} - \frac{d_{33}^2}{\varepsilon_{33}^T}\right)T_3 + \frac{d_{33}Q_f}{\varepsilon_{33}^T A} \tag{14}$$

where $Q_f$ is the charge on a given positive electrode.

To transform this relationship into a deflection control relationship for an entire stack actuator, the following substitutions are made:

$$Q_f = (Q_t/n) \tag{15}$$

$$S_3 = (\Delta l/nt) \tag{16}$$

where $Q_t$ is the total amount of free charge in the $n$ layers of the stack. Note that the stack strain is based on the total amount of active piezoelectric material, rather than the actual stack dimensions, which include dead layers on the stack ends and electrodes between layers. These substitutions result in the following relationship that relates the deflection of the stack actuator, the applied stress, and the total free charge in the stack:

$$\Delta l = nt\left(\frac{1}{Y_{33}} - \frac{d_{33}^2}{\varepsilon_{33}^T}\right)T_3 + \frac{td_{33}Q_t}{\varepsilon_{33}^T A} \tag{17}$$

## Implementation

To evaluate this newly derived control equation [Eq. (17)] it is obvious that a method of delivery of a known amount of free charge to a piezoelectric actuator is needed. Earlier investigations have used charge control of piezoelectric actuators, including Comstock[5] and Newcomb and Flinn,[6] but no models for the charge-deflection behavior were presented. In fact, Newcomb and Flinn argued that there was no reason to expect the relationship between the electric displacement and the strain to be linear.

The charge-feedback circuit concept used by Comstock is shown in Fig. 6. This circuit concept was also used in this investigation. Elementary circuit analysis demonstrates that the charge in the piezoelectric actuator in the circuit obeys the relationship

$$Q_t = CV_{\text{in}} \tag{18}$$

Although this circuit does accomplish the main objective, the delivery of a known charge, it has two readily apparent drawbacks: both sides of the piezoactuator are floating with respect to ground, and this circuit is very sensitive to amplifier bias current; thus it requires some sort of initialization circuit to remove any charge bias between the actuator and series capacitor.

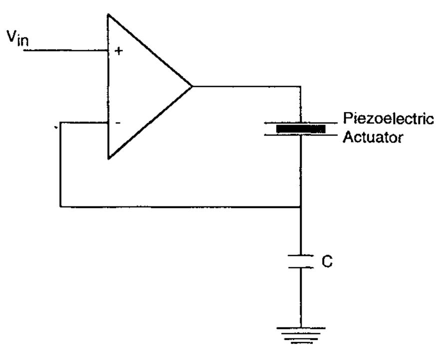


**Fig. 6 Charge-feedback amplifier circuit concept.**

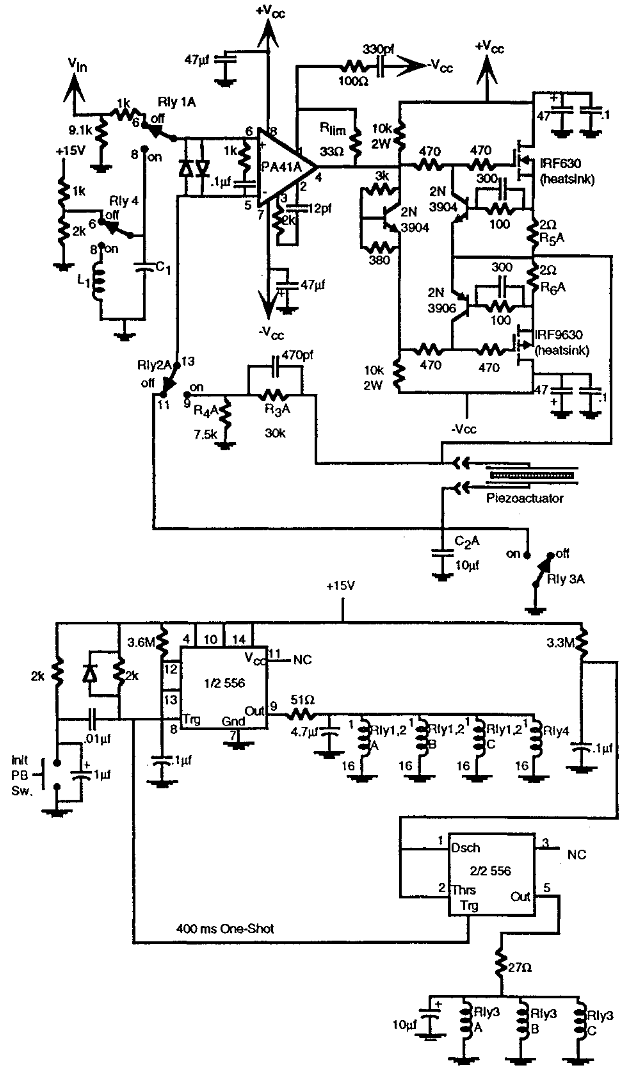


**Fig. 7 Schematic of the charge-feedback amplifier circuit.**

The actual implementation of the circuit concept is shown in Fig. 7. As is typical with real-world electronics, the final design is somewhat more complex than the concept sketch shown in Fig. 6. The amplifier has three major components: an op-amp configured in charge-feedback mode, the initialization circuit, and a current buffer placed at the op-amp output to improve amplifier bandwidth. The initialization circuit applies a damped sinusoidal signal to the op-amp input and thus to the piezoelectric actuator to provide a repeatable actuator zero point. The 556 timer in the initialization circuit is necessary to make sure that relay 3 deactivates about 20 ms before relays 1 and 2. This is to avoid any switching transients being applied to the series capacitor, which will result in a voltage offset. Further details about this amplifier design, with the exception of the current buffer, are reported by Comstock.[5]

## Experimental

Tests of the applied charge-deflection behavior of a piezoelectric stack actuator were undertaken to test the validity of the charge control relationships developed in this paper. The test article consisted of a 15-layer piezoelectric stack actuator made up of wafers of PZT5H. Material properties and other pertinent information on the actuator are listed in Table 1. The stack actuator was inserted into the charge-feedback amplifier circuit as indicated in Fig. 6. The stack was mounted on an optical bench and a metal target was glued to the top of the actuator with cyanoacrylate adhesive

**Table 1 Stack actuator characteristics**

| | |
|---|---|
| Material type | PZT5H |
| Dimensions (L × W × H) | $(4.45 \times 4.45 \times 3.38) \times 10^{-3}$ m |
| $d_{33}$ | $593 \times 10^{-12}$ m/V |
| $\varepsilon^T$ | $2.57 \times 10^{-8}$ f/m |
| $Y_{33}$ | $4.8 \times 10^{10}$ N/m$^2$ |
| $t$ | $1.65 \times 10^{-4}$ m |
| $n$ | 15 |

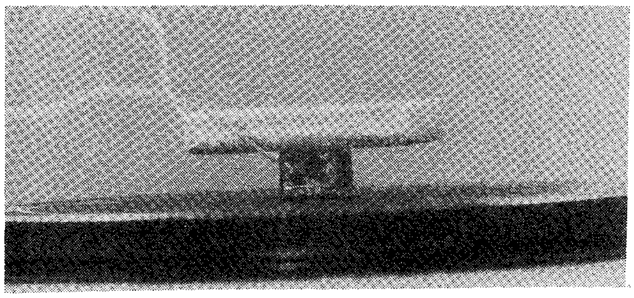

**Fig. 8 Photograph of the cube-shaped piezoelectric stack with the aluminum target plate mounted on top.**

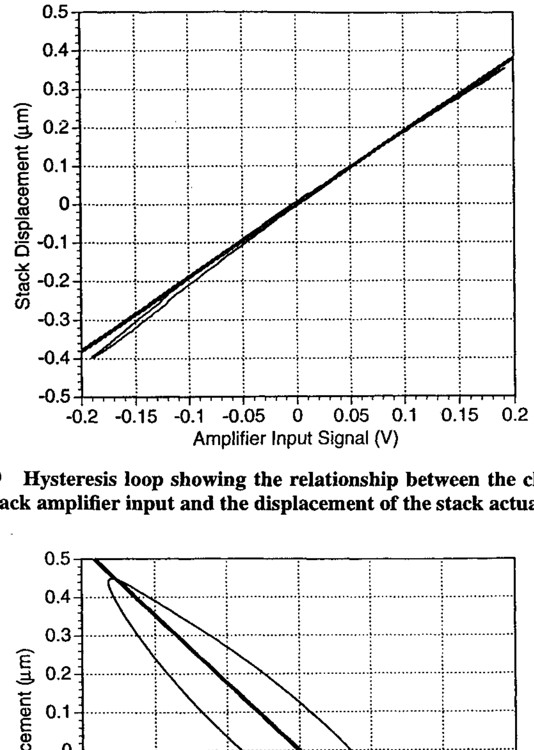


**Fig. 9 Hysteresis loop showing the relationship between the charge-feedback amplifier input and the displacement of the stack actuator.**

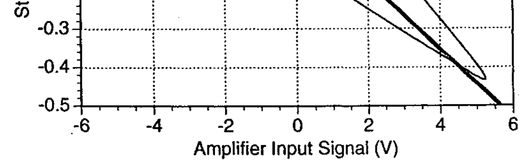


**Fig. 10 Hysteresis loop showing the relationship between the voltage-feedback amplifier input and the displacement of the stack actuator.**

to provide a target for displacement measurement by a capacitance gauge (Fig. 8).

Tests of the amplifier–actuator system consisted of quasistatic (1-Hz) hysteresis loops that covered a stack deflection range of $\pm 0.4$ $\mu$m. The amplifier input range was adjusted to cover this deflection range. Since the tests were quasistatic and there were no other forces applied to the stack actuator during the tests, the quantity $T_3$ in Eq. (17) was taken to be zero. Substitution of this result, as well as Eq. (18) into Eq. (17), yields

$$\Delta l = \frac{t d_{33} C}{\varepsilon_{33}^T A} V_{\text{in}} \tag{19}$$

where $C = 10$ $\mu$f in the charge-feedback amplifier used in this test.

The initialization circuit of the charge-feedback amplifier was engaged at the beginning of each test series. A number of charge-feedback hysteresis tests were performed, but the anticipated problem of voltage drift due to amplifier bias currents was not apparent. In tests lasting occasionally tens of minutes, reinitialization of the circuit to remove any charge bias between the actuator and the capacitor was never necessary. Although no tests were performed specifically to observe actuator drift due to charge accumulation, drift was not a problem in these tests.

Figure 9 shows the hysteresis loops obtained during the tests of the charge feedback amplifier. The gray line accompanying the hysteresis loop has a slope of $t d_{33} C / \varepsilon_{33}^T A$ and represents the prediction of the charge-deflection model embodied in Eq. (17). Shown in Fig. 10 is another hysteresis loop from the same stack actuator under the same test conditions, the difference being that in this case a simple inverting amplifier (gain $= -10$) was used to apply a known voltage to the stack. The line on this plot is the voltage-deflection relationship predicted by Eq. (4). Note the large amount of hysteresis that is present in the voltage-control case, whereas the charge-feedback amplifier case is remarkably free of hysteresis and agrees closely with the model prediction.

## Conclusions

In this investigation the inherent hysteresis in voltage control of piezoelectric actuators has been demonstrated both theoretically and experimentally. This hysteresis makes accurate open-loop control of voltage-driven piezoelectric actuators impossible and complicates closed-loop control, potentially limiting system gains and/or introducing limit cycle behavior. It is also shown that the piezoelectric material constitutive relationships can be reformulated to yield material strain in terms of applied stress and electric displacement, and that this electric displacement term is a controllable quantity.

The geometry of the basic element of most piezoelectric actuators, namely, the flat wafer with distributed electrodes on the top and bottom, makes it possible to apply Gauss' law for dielectric materials to determine the electric displacement. This results in constitutive relationships that depend directly upon the amount of free charge applied to the actuator. Previous works have demonstrated that charge-feedback control reduces actuator hysteresis but did not supply an explicit model for the charge-displacement relationship. Here the relationship between charge and actuator deflection is derived and demonstrated experimentally.

**Table 2 Comparison of voltage-feedback and charge-feedback control**

| | Voltage-feedback control | Charge-feedback control |
|---|---|---|
| Advantages | Power supplies simple and readily available<br>One actuator electrode is grounded | Linear relationship between amplifier input and stack displacement facilitates open-loop control<br>Linear behavior also allows higher system gains in feedback control |
| Disadvantages | Large amounts of hysteresis makes accurate open-loop control impossible<br>Hysteresis also limits gains and may cause limit cycle behavior | Both actuator leads floating with respect to ground (it may be possible to redesign the circuit to avoid this problem)<br>Requires amplifier initialization to remove charge bias |

The significant conclusion of this work is that there now exists a true, predictable alternative to direct voltage control of piezoelectric actuators in the form of charge-feedback control. Which control system is preferable is application dependent (see Table 2). Lower performance systems may respond fine using voltage control, but as applications become more demanding, requiring higher bandwidths and better spatial resolution, the need for improved system response will necessitate directly controlling the charge in a piezoelectric actuator, rather than the voltage applied to it.

## Acknowledgments

The authors gratefully acknowledge the support of the National Science Foundation (Contract MSS9350269) with Dev Garg of the Dynamic Systems and Controls Branch serving as monitor. The authors also acknowledge the support of NASA, particularly Henry Waites at the Marshall Space Flight Center. The design of the charge-feedback amplifier is covered by U.S. Patent 4,263,527 and was used with the permission of the Charles Stark Draper Laboratory, Inc. The current buffer component of the charge-feedback amplifier was designed and built by Chuck Sweet of Garman Systems, Inc.

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